

## nag\_deviates\_normal (g01fac)

### 1. Purpose

**nag\_deviates\_normal (g01fac)** returns the deviate associated with the given probability of the standard Normal distribution.

### 2. Specification

```
#include <nag.h>
#include <nagg01.h>
```

```
double nag_deviates_normal(Nag_TailProbability tail, double p, NagError *fail)
```

### 3. Description

The deviate,  $x_p$  associated with the lower tail probability,  $p$ , for the standard Normal distribution is defined as the solution to:

$$P(X \leq x_p) = p = \int_{-\infty}^{x_p} Z(X) dX$$

where

$$Z(X) = \frac{1}{\sqrt{2\pi}} e^{-X^2/2}, \quad -\infty < X < \infty.$$

The method used is an extension of that of Beasley and Springer (1977).  $p$  is first replaced by  $q = p - 0.5$ .

(a) if  $|q| \leq 0.3$ ,  $x_p$  is computed by a rational Chebyshev approximation

$$x_p = s \frac{A(s^2)}{B(s^2)}$$

where  $s = \sqrt{2\pi} \cdot q$  and  $A$ ,  $B$  are polynomials of degree 7.

(b) if  $0.3 < |q| \leq 0.42$ ,  $x_p$  is computed by a rational Chebyshev approximation

$$x_p = \text{sign } q \left( \frac{C(t)}{D(t)} \right)$$

where  $t = |q| - 0.3$  and  $C$ ,  $D$  are polynomials of degree 5.

(c) if  $|q| > 0.42$ ,  $x_p$  is computed as

$$x_p = \text{sign } q \left\{ \left( \frac{E(u)}{F(u)} \right) + u \right\}$$

where  $u = \sqrt{-2 \times \log(\min(p, 1-p))}$  and  $E$ ,  $F$  are polynomials of degree 6.

For the upper tail probability  $-x_p$  is returned while for the two tail probabilities the value  $x_{p^*}$  is returned where  $p^*$  is the required tail probability computed from the input value of  $p$ .

## 4. Parameters

### tail

Input: indicates which tail the supplied probability represents.

If **tail** = **Nag\_LowerTail**, the lower tail probability, i.e.,  $P(X \leq x_p)$ .

If **tail** = **Nag\_UpperTail**, the upper tail probability, i.e.,  $P(X \geq x_p)$ .

If **tail** = **Nag\_TwoTailSignif**, the two tail (significance level) probability, i.e.,  $P(X \geq |x_p|) + P(X \leq -|x_p|)$ .

If **tail** = **Nag\_TwoTailConfid**, the two tail (confidence interval) probability, i.e.,  $P(X \leq |x_p|) - P(X \leq -|x_p|)$ .

Constraint: **tail** = **Nag\_UpperTail**, **Nag\_LowerTail**, **Nag\_TwoTailSignif** or **Nag\_TwoTailConfid**.

### p

Input: the probability,  $p$ , from the standard Normal distribution as defined by **tail**.

Constraint:  $0.0 < \mathbf{p} < 1.0$ .

### fail

The NAG error parameter, see the Essential Introduction to the NAG C Library.

## 5. Error Indications and Warnings

If **fail.code**  $\neq$  **NE\_NOERROR**, then nag\_deviates\_normal returns 0.0.

### NE\_BAD\_PARAM

On entry, parameter **tail** had an illegal value.

### NE\_REAL\_ARG\_LE

On entry, **p** must not be less than or equal to 0.0: **p** =  $\langle \text{value} \rangle$ .

### NE\_REAL\_ARG\_GE

On entry, **p** must not be greater than or equal to 1.0: **p** =  $\langle \text{value} \rangle$ .

### NE\_INTERNAL\_ERROR

An internal error has occurred in this function. Check the function call and any array sizes.

If the call is correct then please consult NAG for assistance.

## 6. Further Comments

### 6.1. Accuracy

Accuracy is mainly limited by the *machine precision*.

### 6.2. References

Abramowitz M and Stegun I A (1972) *Handbook of Mathematical Functions* Ch. 7.1, p.297 and Ch. 26.2, p. 931 Dover Publications, New York.

Beasley J D and Springer S G (1977) Algorithm AS111. The Percentage Points of the Normal Distribution *Appl. Statist.* **26** 118–120.

Hastings N A J and Peacock J B (1977) *Statistical Distributions* Ch. 21, pp.96–101 Butterworth.

## 7. See Also

None

## 8. Example

Four values of **tail** and **x** are input and the probabilities calculated and printed.

### 8.1. Program Text

```
/* nag_deviates_normal(g01fac) Example Program.
 *
 * Copyright 1996 Numerical Algorithms Group.
 *
 * Mark 4, 1996.
 *
```

```

*/

#include <nag.h>
#include <stdio.h>
#include <nag_stdlib.h>
#include <nagg01.h>

main()
{
    double p;
    double dev;
    Integer i;
    char tail_char;
    Nag_TailProbability tail;

    Vprintf(" g01fac Example Program Results\n");
    /*      Skip heading in data file */
    Vscanf("%*[^\\n] ");
    Vprintf("\n Tail      Probability      Deviate \\n\\n");
    for (i = 1; i <= 4; ++i)
    {
        Vscanf("%c %lf ", &tail_char, &p);
        switch (tail_char)
        {
            case 'L':
                tail=Nag_LowerTail;
                break;
            case 'U':
                tail=Nag_UpperTail;
                break;
            case 'C':
                tail=Nag_TwoTailConfid;
                break;
            case 'S':
                tail=Nag_TwoTailSignif;
        }

        dev = g01fac(tail, p, NAGERR_DEFAULT);
        Vprintf("      %c          %5.3f          %6.4f\\n", tail_char, p, dev);
    }
    exit(EXIT_SUCCESS);
}

```

## 8.2. Program Data

g01fac Example Program Data  
L 0.975  
U 0.025  
C 0.95  
S 0.05

## 8.3. Program Results

g01fac Example Program Results

Tail	Probability	Deviates
L	0.975	1.9600
U	0.025	1.9600
C	0.950	1.9600
S	0.050	1.9600

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